R & D NOTES

A Simple Method for Safety Factor Evaluation

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Accurate design of process equipment for a required capacity is often unreliable owing to uncertainty of the design data. Therefore, overdesign is often used to increase reliability. This is commonly done by employing conventional safety factors that are based on previous experience. (Rudd and Watson, 1968).

More sophisticated methods for evaluating safety factors have been suggested. These methods are based on probability calculations (Saletan and Caselli, 1963; Saletan, 1969; Freeman and Gaddy, 1975), expected cost minimization (Kittrel and Watson, 1966; Wen, 1968), and stochastic simulation (Lashmet and Szczepanski, 1974).

Practically, these methods are limited because their use entails prior knowledge of the statistical distribution functions of the uncertain parameters. Often, this information is not available, and the data are characterized only by their expected value and standard error.

This note describes a short-cut method, which is independent of statistical distribution, for estimation of required overdesign.

OVERDESIGN CRITERIA

An appropriate criterion is required for estimating the safety factors. Optimization of expected costs (Kittrel and Watson, 1966; Wen, 1968) does not seem to fit the case under discussion because this criterion fails to account for the influence of the applied safety factor on the reliability of design. On the other hand, probability calculations (Saletan and Caselli, 1963; Saletan, 1969; Freeman and Gaddy, 1975) relate these two important subjects and will be used hereafter.

In particular, the safety first principle (Roy, 1950) is referred to. This principle states that the probability of failure should not exceed a preset value.

THE MATHEMATICAL MODEL

The concept of overdesign can be described in statistical terms by assuming that the actual capacity of the equipment is a random variable, being a function of uncertain data.

Denoting the actual capacity by c and the required capacity by d, we can define the design reliability R by

$$R = Pr(c \geq d) \qquad (1)$$

In order to meet specified capacity reliability requirements, the equipment must be designed for a capacity $\bar{c}$ which is higher than d. The safety factor is thus defined by the ratio

$$SF = \frac{\bar{c}}{d} \qquad (2)$$

The statistical model implies that the designed capacity $\bar{c}$ is the expectation of the actual capacity c, and the required capacity d is its low-tail value of confidence level R.

This situation is shown in Figure 1 for an arbitrary distribution function.

Assume that the uncertainty in estimating equipment capacity is estimated by σ_c^2, the variance of the capacity. A relationship between σ_c and $\bar{c}$ can be evaluated from the standard errors of the data and the design equations by common statistical methods (Wine, 1964).

The relationship between the uncertainty of the performance of an entire process and the design reliability
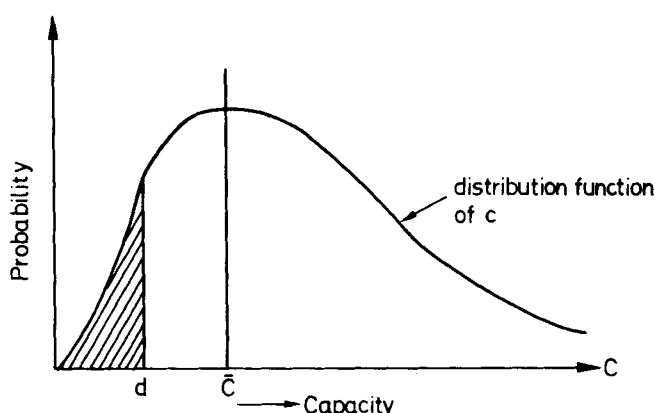

Fig. 1. The statistical model of overdesign and reliability. (Reliability = unshaded area.)
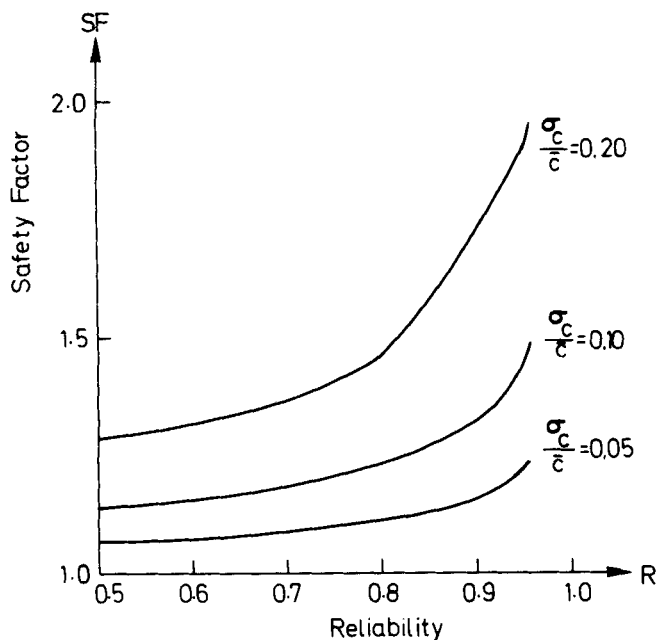
Fig. 2. Sample calculations of safety factors: $(\sigma_c/\bar{c} = 0.2)$.
(1) General (bound)
(2) Unimodal symmetrical (bound)
(3) Normal distribution

of its elements can be obtained by probability calculations. These topics are outside the scope of this note; they are discussed by Saleton (1963, 1969).

EVALUATION OF SAFETY FACTORS

If we assume that the distribution function of c is unknown, it is impossible to formulate an exact relationship between safety factor and reliability. An alternative is to estimate their bounds.

This can be done by applying the Tshebishyev inequality (Wine, 1964) to the statistical model of design uncertainty:

$$Pr[|c - \bar{c}| \geq t] \leq \frac{\sigma_c^2}{t^2} \qquad (3)$$

where t is a positive constant.

Setting $t = \bar{c} - d$, we get

$$Pr[|c - \bar{c}| \geq (\bar{c} - d)] \leq \frac{\sigma_c^2}{(\bar{c} - d)^2} \qquad (4)$$

Because only the range $\bar{c} \geq c$ is significant, this inequality can be rewritten as

$$Pr[\bar{c} - c \geq \bar{c} - d] \leq \frac{\sigma_c^2}{(\bar{c} - d)^2} \qquad (5)$$

or as

$$Pr[d \geq c] \leq \frac{\sigma_c^2}{(\bar{c} - d)^2} \qquad (6)$$

According to the definition, we get

$$Pr[d \geq c] = 1 - Pr[c \geq d] = 1 - R \qquad (7)$$

By substituting Equation (7) and the definition of the safety factor [Equation (2)] into inequality (6), one obtains (after rearrangement of terms)

$$SF \leq 1 + \frac{1}{\sqrt{1 - R}} \frac{\sigma_c}{\bar{c}} \qquad (8)$$

This inequality sets an upper bound on the required safety factor as function of the desired design reliability and the relative uncertainty of the capacity.

Smaller limits of overdesign are obtained by assuming the distribution of the capacity to be unimodal and symmetrical. In this case (which may be realistic) another inequality (Roy, 1950) can be used:

$$Pr[d \geq c] \leq \frac{4}{9} \cdot \frac{\sigma_c^2}{(\bar{c} - d)^2} \qquad (9)$$

This inequality is rearranged for relating reliability to overdesign, and the following result is obtained:

$$SF \leq 1 + \frac{2/3}{\sqrt{1 - R}} \cdot \frac{\sigma_c}{\bar{c}} \qquad (10)$$

This case requires overdesign limits that are 33% lower than the general case of completely unknown distribution.

It is clear that the sharpest results are obtained when the distribution function is known. Sample calculations of safety factors according to inequalities (8) and (10) are shown in Figure 2 and compared to safety factors which are based on normally distributed capacity.

DISCUSSION

The statistical interpretation of overdesign and design reliability of equipment capacity with uncertain design data is discussed. The Tshebishyev inequality is used to evaluate bounds of required safety factors for an unknown statistical distribution as a function of the desired reliability. Bounds of safety factors are calculated also for the less general (but common) case of an unimodal and symmetrical distribution function. For comparison, a normally distributed capacity was also considered.

It is shown that safety factors which are based on an unknown distribution function are higher than those required for more defined cases. Therefore, if cost of overdesign is high, the use of more sophisticated methods for evaluating safety factors, such as simulation, is to be considered.

The optimal values of the design reliability and required overdesign should be determined by considering cost of failure against cost of oversizing. Such calculations were demonstrated by Saleton and Caselli (1963) and Saleton (1969).

NOTATION

c	= actual capacity
$\bar{c}$	= expected capacity
d	= required minimal capacity
OD	= amount of overdesign
Pr	= probability
R	= reliability of design
SF	= safety factor
t	= arbitrary constant (in Tshebishyev inequality)
σ_c	= standard error of the capacity

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The Role of Vapor Phase Composition in Data Reduction of Vapor-Liquid Equilibria

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For vapor-liquid equilibrium (VLE) data which are characterized by insufficient consistency, there is only one poor agreement between the excess Gibbs function $G^E_{(P,y)}$ derived from P-x-y data and $G^E_{(P)}$ calculated by an integration procedure without using y data. As pointed out in earlier publications, in these cases the $G^E_{(P)}$ values should be preferred (Kehlen, 1969; Kehlen and Peil, 1972; Van Ness et al., 1973; Kuschel et al., 1974; Prausnitz, 1975; Van Ness, 1975). Therefore, efforts in experimental work should be directed to achieve more accurate P-x values and not to determine P-x-y simultaneously. To support this conclusion, this note discusses some selected examples and presents calculations with model systems.

TREATMENT OF DATA

Calculation of the molar excess Gibbs function G^E from isothermal P-x-y data is performed by the conventional procedure. Because of the Gibbs-Duhem equation, there exists also the possibility of calculating G^E only from P-x data. For this purpose we used the procedure developed by Barker (1953).

To test the consistency of data measured at n different mole fractions x_i, the consistency parameter K is calculated. K is defined by

$$K = \frac{\sum_{i=1}^{n} |G^E_{(P,y)}(x_i) - G^E_{(P)}(x_i)|}{n} \quad (1)$$

If the measurements (and the data reduction) are carried out without errors, then $G^E_{(P,y)}$ and $G^E_{(P)}$ must be identical ($K = 0$). But in real systems, no total agreement occurs, indicating that there is no perfect consistency; hence $K > 0$. For good consistency, the requirement is $K \leqslant 15$ J/mole (Kehlen et al., 1969).

In principle, one can achieve sufficient agreement by improving the accuracy of the measurements. But this is often very difficult, especially in systems with high rela-

tive volatility. Thus, in the case of inconsistency, the question arises whether $G^E_{(P)}$ or $G^E_{(P,y)}$ possesses the higher reliability.

DISCUSSION OF SELECTED EXAMPLES

For the system carbon disulfide (1) + acetone (2), the following data exist:

(a) v.Zawidski (1900):
    35.17°C; P-x-y data; $K = 15.5$ J/mole

(b) Hirshberg (1932):
    29.2°C; P-x-y data; $K = 56.2$ J/mole

(c) Schmidt (1926):
    35 °C; P-x data

Figure 1 shows the result of data reduction for the cases (a) and (b). Good agreement is obtained between $G^E_{(P)}$ (a), $G^E_{(P,y)}$ (a), and $G^E_{(P)}$ (b). There is also good agreement with the $G^E_{(P)}$ data based on the measurements of Schmidt (1926). Only the $G^E_{(P,y)}$ values, calculated from the measurements of Hirshberg (1932), indicate bad consistency and do not agree with the other results.

Brown (1952) has reported P-x-y measurements for the system cyclohexane + nitrobenzene at 80°C. From his results we obtained $G^E_{(P,y)} \approx 1\,640$ J/mole; $G^E_{(P)} \approx 1\,150$ J/mole ($x = 0.5$). Brown (1952) explained the poor consistency of his results ($K = 430$ J/mole) by experimental difficulties (relative volatility $\alpha = 105$). Mecke and Zirker (1964) studied the liquid-liquid and the liquid-solid equilibrium of this system as well as the heat of mixing. Their measurements yield at $x = 0.5$; $G^E \approx 1\,250$ J/mole (at -2.5°C) and $H^E \approx 1\,680$ J/mole (at 20°C). These data permit the calculation of G^E at 80°C: $G^E \approx 1\,105$ J/mole ($x = 0.5$). This value shows good agreement with $G^E_{(P)}$ of Brown (1952) but very bad agreement with $G^E_{(P,y)}$.

Udovenko and Frid (1948) reported sets of P-x-y data for the five binary systems methanol + 2-methyl-propan-1-ol, methanol + 3-methyl-butan-1-ol, ethanol + propan-1-ol, ethanol + 2-methyl-propan-1-ol, and ethanol + 3-methyl-butan-1-ol at 50°, 60°, and 70°C. For most of the measurements, the thermodynamic consistency is insufficient ($K > 15$ J/mole). Table 1 summarizes $G^E_{(P,y)}$ and $G^E_{(P)}$ as well as the excess enthalpies determined by Hirobe (1925) at 25°C.

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